

# Color superconductivity in dense quark matter

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## Abstract

I discuss recent developments in our understanding of the color-superconducting phases of cold, dense quark matter. I describe the phase diagram as a function of density and the strange quark mass, and outline some ideas about possible observational consequences of these exotic phases.

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# 1 Introduction

The phase diagram of QCD is a topic of intensive research, both theoretical and experimental. In recent years, the high-temperature low-density regime has been studied in lattice gauge calculations, and probed in heavy-ion experiments [1]. The low-temperature high-density regime has remained more mysterious, because lattice calculations run into intractable problems at non-zero chemical potential, and heavy-ion collision experiments are focussing on high temperature in order to definitively produce the quark-gluon plasma. Even so, cold dense quark matter is of direct physical relevance (in neutron stars, for example), and recently there has been striking theoretical progress in our understanding of its symmetries and basic properties. In this paper I will review some of the elements of the new picture that is emerging.

At high densities and low temperatures, matter consists of a Fermi sea of quarks. The relevant degrees of freedom are those which involve quarks with momenta near the Fermi surface. These interact via gluons, in a manner described by QCD. The quark-quark interaction has two channels available, the antisymmetric  $\mathbf{\bar{3}}$ , and the symmetric  $\mathbf{6}$ . It is attractive in the  $\mathbf{\bar{3}}$ : this can be seen from single-gluon-exchange, or by counting of strings, or from the 'tHooft vertex induced by instantons.

It was shown by Bardeen, Cooper, and Schrieffer (BCS) [2] that a Fermi surface in the presence of attractive interactions is unstable. If there is *any* channel in which the quark-quark interaction is attractive, then the true ground state of the system will not be the naked Fermi surface, but rather a condensate of quark Cooper pairs.

This can be seen intuitively as follows. Consider a system of free particles. The Helmholtz free energy is  $F = E - \mu N$ , where  $E$  is the total energy of the system,  $\mu$  is the chemical potential, and  $N$  is the number of particles. The Fermi surface is defined by a Fermi energy  $E_F = \mu$ , at which the free energy is minimized, so adding or subtracting a single particle costs zero free energy. Now, suppose a weak attractive interaction is switched on. BCS showed that this does not simply lead to a small shift in the Fermi energy. Rather, it leads to a complete rearrangement of the states near the Fermi surface. The mechanism is simple: it costs no free energy to make a pair of particles (or holes), and because of the attractive interaction it is favorable to do so. Many such pairs will therefore be created, in all the modes near the Fermi surface, and these pairs, being bosonic, will form a condensate.

The ground state will be a superposition of states with all numbers of pairs, breaking the fermion number symmetry. An arbitrarily weak interaction has lead to spontaneous symmetry breaking.

In condensed matter systems, the BCS mechanism leads to superconductivity, since it causes Cooper pairing of electrons, which breaks the electromagnetic gauge symmetry, giving mass to the photon and producing the Meissner effect (exclusion of magnetic fields from a superconducting region). It is a rare and delicate state, easily disrupted by thermal fluctuations, because the dominant interaction between electrons is the repulsive electrostatic force, and only in the right kind of crystal are there attractive phonon-mediated interactions that can overcome it.

In QCD, by contrast, the dominant gauge-boson-mediated interaction between quarks is itself attractive. This means that we expect quark pairing to happen whenever the density becomes large enough for the top of the Fermi sea to consist of quarks rather than nucleons (“quark matter”). Since pairs of quarks cannot be color singlets, the resulting condensate will break the local color symmetry  $SU(3)_{\text{color}}$ . We call this “color superconductivity”. Note that the quark pairs play the same role here as the Higgs particle does in the standard model: the color-superconducting phase can be thought of as the “Higgsed” (as opposed to “confined”) phase of QCD.

It is important to remember that the breaking of a gauge symmetry cannot be characterized by a gauge-invariant local order parameter which vanishes on one side of a phase boundary. The superconducting phase of QCD can be characterized rigorously only by its global symmetries. In electromagnetism there is a non-local order parameter, the photon magnetic mass (Meissner effect), that distinguishes the free phase from the superconducting one. In QCD there is no free phase: even without pairing the gluons are not states in the spectrum. No order parameter distinguishes the Higgsed phase from a confined phase or a plasma, so we have to look at the global symmetries.

In this paper we will study QCD with 2+1 quarks. We take the up and down quarks to be massless, and study the phase structure of cold, dense QCD as a function of the strange quark mass. We will start with the limiting cases of two or three massless quarks.

Table 1: Symmetries of phases of QCD. With 2 massless flavors, the quark paired phase has the same symmetries as the QGP, but is different from nuclear matter. With 3 massless flavors, the quark paired phase has the same symmetries as the hypernuclear matter phase, but is different from the QGP. See Fig. 1, 2 and 3.  $I_3$  is an isospin generator,  $T_8$  is a color generator.

phase	electromagnetism	chiral symmetry	baryon number
QGP	$Q$	unbroken	$B$
2 flavor nuclear matter	broken	broken	broken
2 flavor quark pairing (2SC)	$\tilde{Q} = Q - \frac{1}{2\sqrt{3}}T_8$	unbroken	$\tilde{B} = \tilde{Q} + I_3$
3 flavor nuclear matter	$Q$	broken	broken
3 flavor quark pairing (CFL)	$\tilde{Q} = Q + \frac{1}{\sqrt{3}}T_8$	broken	broken

## 2 Two or three massless flavors

In QCD with two flavors of massless quarks the Cooper pairs form in the color  $\bar{\mathbf{3}}$  flavor singlet channel [3, 4, 5, 6, 7, 8]

$$\langle q_i^\alpha q_j^\beta \rangle \sim \varepsilon_{ij} \varepsilon^{\alpha\beta 3} \quad (2.1)$$

The pattern of symmetry breaking is therefore (with gauge symmetries in square brackets)

$$\begin{aligned} & [SU(3)_{\text{color}}] \times [U(1)_Q] \times SU(2)_L \times SU(2)_R \\ \rightarrow & [SU(2)_{\text{color}}] \times [U(1)_{\tilde{Q}}] \times SU(2)_L \times SU(2)_R \end{aligned} \quad (2.2)$$

The resulting condensate gives gaps to quarks with two out of the three colors, and breaks the local color symmetry  $SU(3)_{\text{color}}$  to an  $SU(2)_{\text{color}}$  subgroup. The Cooper pairs are  $ud - du$  flavor singlets and, in particular, the global flavor  $SU(2)_L \times SU(2)_R$  symmetry is left intact. There is an unbroken global symmetry which plays the role of baryon number symmetry,  $U(1)_B$ . Thus, no global symmetries are broken and the only putative Goldstone bosons are those five which become the longitudinal parts of the five gluons

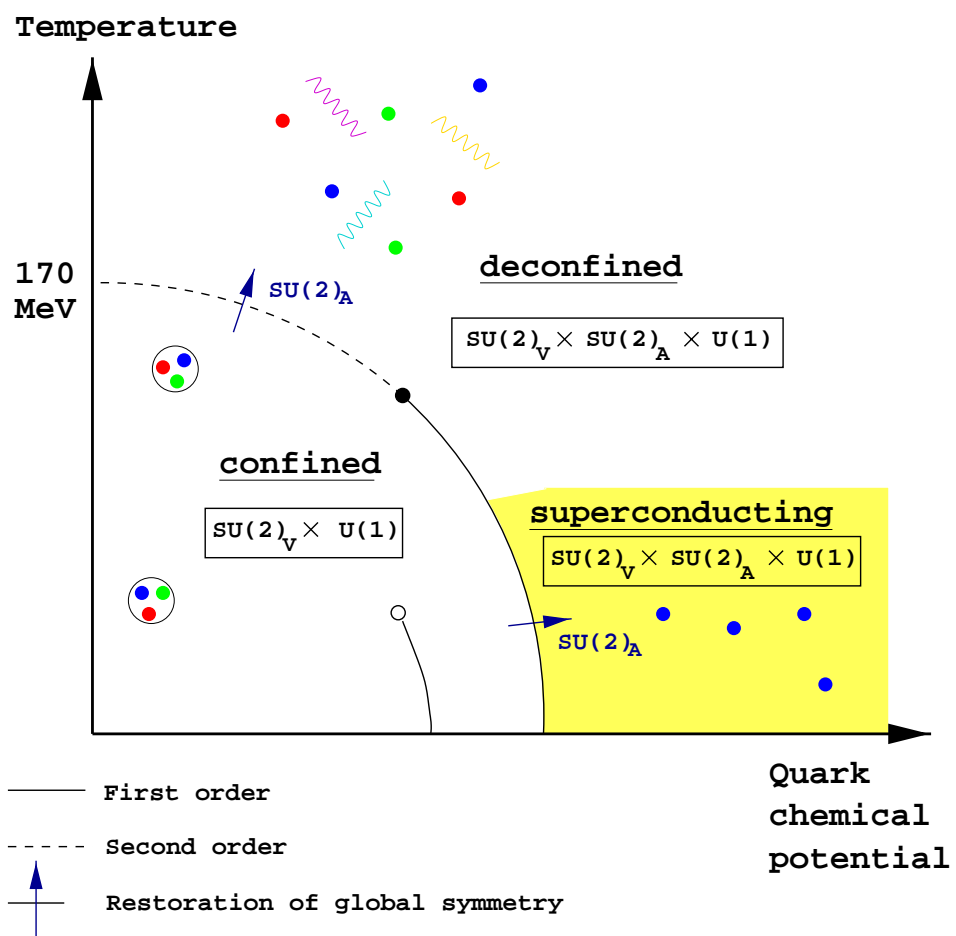


Figure 1: Two massless flavor phase diagram

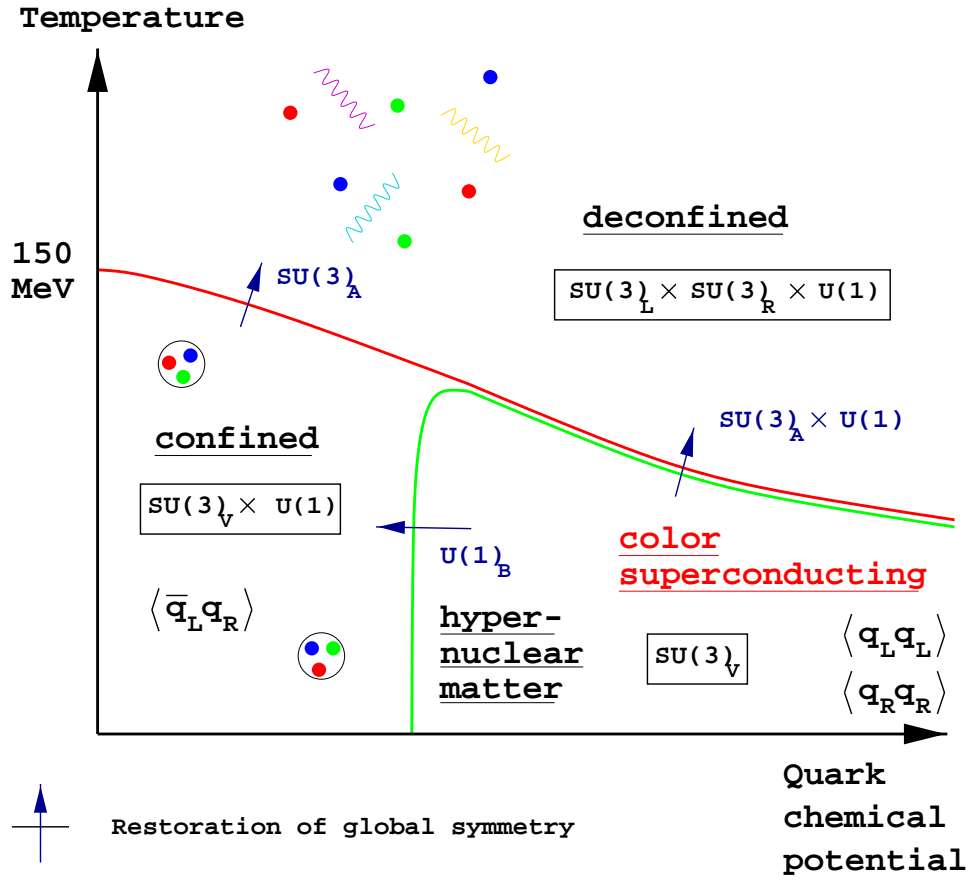


Figure 2: Three massless flavor phase diagram

which acquire masses [6]. There is also an unbroken gauged symmetry which plays the role of electromagnetism.

The third color is prevented from condensing by 'tHooft anomaly matching conditions [9], which require that there be massless fermions if chiral symmetry is unbroken.

In QCD with three flavors of massless quarks the Cooper pairs *cannot* be flavor singlets, and both color and flavor symmetries are necessarily broken. The symmetries of the phase which results have been analyzed in Ref. [10] (see also [11] in which this ordering was studied at zero density). The attractive channel favored by one-gluon exchange exhibits “color-flavor locking”

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta + \kappa \delta_j^\alpha \delta_i^\beta \quad (2.3)$$

This ansatz is only invariant under equal and opposite color and flavor rotations, and since only the vectorial part of color is a symmetry of QCD, only the vectorial part of the flavor group leaves the ansatz invariant. This pattern of pairing therefore breaks chiral symmetry.

$$\begin{aligned} & [SU(3)_{\text{color}}] \times [U(1)_Q] \times SU(3)_L \times SU(3)_R \times U(1)_B \\ \rightarrow & [U(1)_{\tilde{Q}}] \times SU(3)_{C+L+R} \end{aligned} \quad (2.4)$$

There an unbroken gauged  $U(1)$  symmetry (under which all quarks have integer charges) which plays the role of electromagnetism. All nine quarks have a gap. All eight gluons get a mass. There are nine massless Nambu Goldstone excitations of the condensate of Cooper pairs which result from the breaking of the axial  $SU(3)_A$  and baryon number  $U(1)_B$ . We see that cold dense quark matter has rather different global symmetries for  $m_s = 0$  than for  $m_s = \infty$ .

### 3 2+1 flavor dense matter

A nonzero strange quark mass explicitly breaks the flavor  $SU(3)_V$  symmetry. As a consequence, color-flavor locking with an unbroken global  $SU(3)_{\text{color}+V}$  occurs only for  $m_s \equiv 0$ . Instead, for nonzero but sufficiently small strange quark mass we expect color-flavor locking which leaves a global  $SU(2)_{\text{color}+V}$  group unbroken. As  $m_s$  is increased from zero to infinity, there has to be some value  $m_s^{\text{unlock}}$  at which color and flavor rotations are unlocked, and the full  $SU(2)_L \times SU(2)_R$  symmetry is restored. It can be argued on general grounds (Sect. 4) that such a simple unlocking phase transition must be first

order, although it is also possible that there may be a crystalline intermediate phase [12, 13].

An analysis of the unlocking transition, using a NJL model with interaction based on single-gluon exchange [14], indicates that for realistic values of the strange quark mass chiral symmetry breaking may be present for densities all the way down to those characteristic of baryonic matter. This raises the possibility that quark matter and baryonic matter may be continuously connected in nature, as Schäfer and Wilczek have conjectured for QCD with three massless quarks [15]. The gaps due to pairing at the quark Fermi surfaces map onto gaps due to pairing at the baryon Fermi surfaces in superfluid baryonic matter consisting of nucleons,  $\Lambda$ 's,  $\Sigma$ 's, and  $\Xi$ 's. (See Section 6).

Color-flavor locking will always occur for sufficiently large chemical potential, for any nonzero, finite  $m_s$  (see Section 7. As a consequence of color-flavor locking, chiral symmetry is spontaneously broken even at asymptotically high densities, in sharp contrast to the well established restoration of chiral symmetry at high temperature.

Figure 3 summarizes our conjecture for the zero temperature phase diagram of QCD as a function of current strange quark mass  $m_s$  and quark number chemical potential  $\mu$ . (we will refer to the  $\mu$ -dependent constituent quark mass as  $M_s(\mu)$ ).

Lines in the diagram separate phases which differ in their global symmetries. In each region of the diagram, we list the unbroken global symmetries of the corresponding phase. We characterize the phases using the  $SU(2)_L \times SU(2)_R$  flavor rotations of the light quarks, and the  $U(1)_S$  rotations of the strange quarks. The  $U(1)_B$  symmetry associated with baryon number is a combination of  $U(1)_S$ , a  $U(1)$  subgroup of isospin, and the gauged  $U(1)_{\text{EM}}$  of electromagnetism. Therefore, in our analysis of the global symmetries, once we have analyzed isospin and strangeness, considering baryon number adds nothing new.

In Fig. 3 we neglect the small  $u$  and  $d$  current quark masses, since they have little effect on the condensation of quark Cooper pairs [8, 16]. We also ignore the effects of electromagnetism. We assume that wherever a baryon Fermi surface is present, baryons always pair at zero temperature. To simplify our analysis, we assume that baryons always pair in channels which preserve rotational invariance, breaking internal symmetries such as isospin if necessary.

We can understand Fig. 3 by considering the phase transitions which



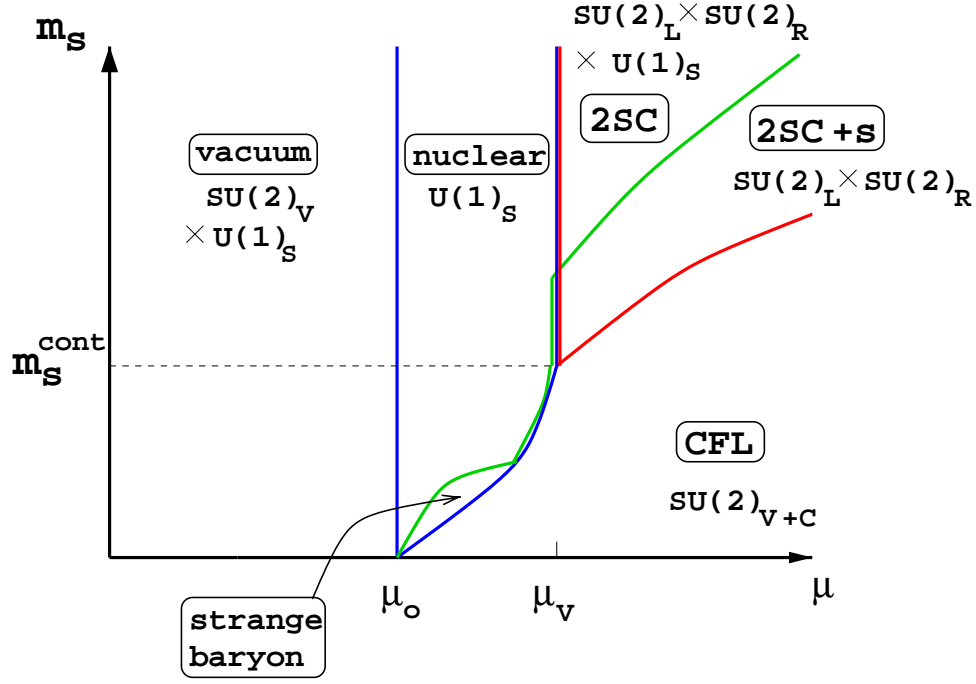


Figure 3: Conjectured phase diagram for QCD with two massless quarks and a strange quark at zero temperature. The global symmetries of each phase are labelled. The regions of the phase diagram labelled 2SC, 2SC+s and CFL denote color superconducting quark matter phases. A detailed explanation is given in the text.

occur as  $\mu$  is increased from 0 to  $\infty$  at constant  $m_s$  for both large and small  $m_s$ .

### 3.1 Heavy strange quark

Assume the strange quark is heavy enough that immediately above deconfinement  $\mu$  is still less than  $m_s$ , so there are still no strange quarks present. For  $\mu = 0$  the density is zero; isospin and strangeness are unbroken; Lorentz symmetry is unbroken; chiral symmetry is broken.

Above a first order transition [17] at an onset chemical potential  $\mu_o \sim 300$  MeV, one finds nuclear matter. Lorentz symmetry is broken, leaving only rotational symmetry manifest. Chiral symmetry is thought to be broken,

although the chiral condensate  $\langle \bar{q}q \rangle$  is expected to be reduced from its vacuum value. In the nuclear matter phase there is  $pp$  and  $nn$  pairing at their Fermi surfaces, breaking isospin. Since there are no strange baryons present,  $U(1)_S$  is unbroken.

When  $\mu$  is increased above  $\mu_V$ , a nucleon description is no longer appropriate. We find the “2SC” phase of color-superconducting quark matter consisting of up and down quarks only, described in Refs. [3, 4, 5, 6, 7]. The light quarks pair in isosinglet Lorentz singlet channels. The full chiral flavor symmetry  $SU(2)_L \times SU(2)_R$  is unbroken. The phase transition at  $\mu_V$  is first order [6, 7, 8, 16, 18, 19] and is characterized by a competition between the chiral  $\langle \bar{q}q \rangle$  condensate and the superconducting  $\langle qq \rangle$  condensate [8, 19].

As the chemical potential is increased further, when  $\mu$  exceeds the constituent strange quark mass  $M_s(\mu)$  a strange quark Fermi surface forms, with a Fermi momentum far below that for the light quarks. We denote the resulting phase “2SC+s”. Light and strange quarks do not pair with each other, because their respective Fermi momenta are so different (see Section 4). The strange Fermi surface is presumably nevertheless unstable. (For more on single flavor pairing, see Ref. [20]). The resulting  $ss$  condensate is expected to be small [14], so we neglect the difference between the 2SC and 2SC+s phases.

Finally, when the chemical potential is high enough that the Fermi momenta for the strange and light quarks become comparable, we pass through the first order locking transition and find the color-flavor locked (CFL) phase. There is an unbroken global symmetry constructed by locking the  $SU(2)_V$  isospin rotations to an  $SU(2)$  subgroup of color. Chiral symmetry is once again broken.

### 3.2 Light strange quark

We now describe the sequence of phases which arise as  $\mu$  is increased, this time for a value of  $m_s$  small enough that strange baryonic matter forms below the deconfinement density. At  $\mu_o$ , one enters the nuclear matter phase, with the familiar  $nn$  and  $pp$  pairing that breaks isospin. The  $\Lambda$ ,  $\Sigma$  and  $\Xi$  densities are still zero, and strangeness is unbroken. At a somewhat larger chemical potential, we enter the strange baryonic matter phase, with a strange baryon Fermi surface (presumably for the  $\Lambda$  or  $\Sigma$ , self-pairing in a spin singlet) breaking  $U(1)_S$ . This phase is labelled “strange baryon” in Figure 3. The global symmetries  $SU(2)_L \times SU(2)_R$  and  $U(1)_S$  are all broken. As  $\mu$  rises,

one may find other onsets at which some of the remaining strange baryon densities become non-zero. These break no new symmetries, and so are not shown in the figure. Note that kaon condensation [21] breaks  $U(1)_S$ , and  $SU(2)_V$ , and so by definition it occurs within the "strange baryon" region of the diagram. If kaon condensation is favored, this will tend to enlarge that region.

We can imagine two possibilities for what happens next as  $\mu$  increases further. (1) Deconfinement: the baryonic Fermi surface is replaced by  $u, d, s$  quark Fermi surfaces, which are unstable against pairing, and we enter the CFL phase. Isospin is locked to color and  $SU(2)_{\text{color}+V}$  is restored, but chiral symmetry remains broken. (2) No deconfinement: the Fermi momenta of all of the octet baryons are now similar enough that pairing between baryons with differing strangeness becomes possible. At this point, isospin is restored: the baryons pair in rotationally invariant, electromagnetically neutral, isosinglets ( $p\Xi^-$ ,  $n\Xi^0$ ,  $\Sigma^+\Sigma^-$ ,  $\Sigma^0\Sigma^0$ ,  $\Lambda\Lambda$ ). The interesting point is that scenario (1) and scenario (2) are indistinguishable in their symmetries. Both look like the "CFL" phase of the figure:  $U(1)_S$  and chirality are broken, and there is an unbroken vector  $SU(2)$ . This is the "continuity of quark and hadron matter" described by Schäfer and Wilczek [15]. We conclude that for low enough strange quark mass,  $m_s < m_s^{\text{cont}}$ , there may be a region where sufficiently dense baryonic matter has the same symmetries as quark matter, and there need not be any phase transition between them. In Section 6 we use this observation to construct a mapping between the gaps we have calculated at the Fermi surfaces in the quark matter phase and gaps at the baryonic Fermi surfaces at lower densities.

## 4 Model-independent features of the unlocking phase transition

Assuming that no other intermediate phases are involved, we can give a model-independent argument that the unlocking phase transition between the CFL and 2SC phases in Figure 3 must be first order.

A strange quark mass explicitly breaks the  $SU(3)$  flavor symmetry down to the  $SU(2)$  involving only the  $u$  and  $d$  quarks. The CFL and 2SC phases are therefore distinguished by whether the chiral  $SU(2)_A$  rotations are spontaneously broken. The unlocking transition is associated with the vanishing of those diquark condensates which pair a strange quark with either

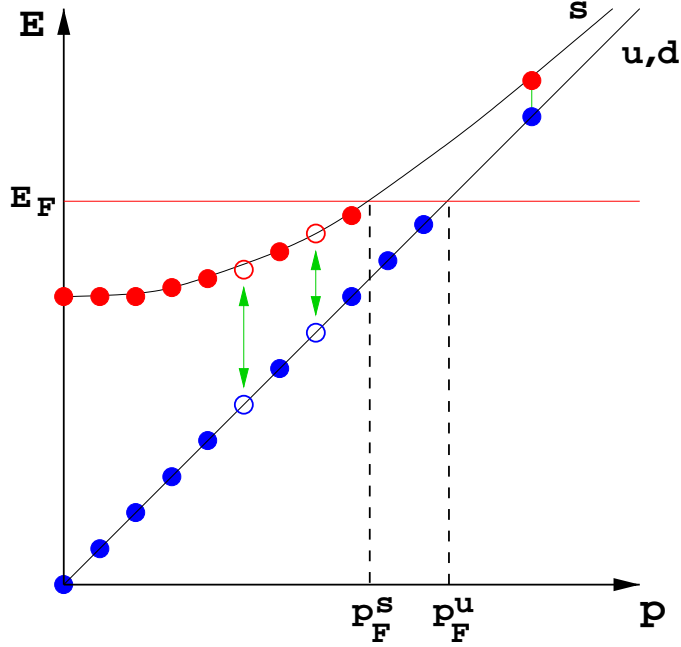


Figure 4: How the strange quark mass disrupts a  $u$ - $s$  condensate. The strange quark (upper curve) and light quark (straight line) dispersion relations are shown, with their Fermi seas filled up to the Fermi energy  $E_F$ . The horizontal axis is the magnitude of the spatial momentum; pairing occurs between particles (or holes) with the same  $p$  and opposite  $\vec{p}$ . For  $p < p_F^s$ , hole-hole pairing ( $\bar{s}\bar{u}$ ) is possible (two examples are shown). For  $p > p_F^u$ , particle-particle pairing ( $s$ - $u$ ) is possible (one example is shown). Between the Fermi momenta for the  $s$  and  $u$  quarks, no such pairing is possible.

an up or a down quark. We denote the resulting gaps  $\Delta_{us}$  for simplicity. In the absence of any  $\Delta_{us}$  gap, the only Cooper pairs are those involving pairs of light quarks, or pairs of strange quarks. The light quark condensate is unaffected by the strange quarks, and behaves as in a theory with only two flavors of quarks. Chiral symmetry is unbroken. When  $\Delta_{us} \neq 0$ , the interaction between the light quark condensates and the mixed (light and strange) condensate results in the breaking of two-flavor chiral symmetry  $SU(2)_A$  via the locking of  $SU(2)_L$  and  $SU(2)_R$  flavor symmetries to an  $SU(2)$  subgroup of color. This color-flavor locking mechanism leaves a global  $SU(2)_{\text{color}+V}$  group unbroken.

The unlocking transition is a transition between a phase with  $\Delta_{us} \neq 0$  at  $M_s < M_s^{\text{unlock}}$  and a phase with  $\Delta_{us} = 0$ . ( $M_s(\mu)$  is the constituent strange quark mass at chemical potential  $\mu$ .) The BCS mechanism guarantees superconductivity in the presence of an arbitrarily weak attractive interaction, and there is certainly an attraction between  $u$  and  $s$  quarks (with color  $\mathbf{\bar{3}}$ ) for any  $M_s$ . Naively, then, it would seem impossible for  $\Delta_{us}$  to vanish above  $M_s^{\text{unlock}}$ . The BCS result relies on a singularity which arises for pairs of fermions with zero total momentum *at* the Fermi surface. We see from Figure 4 that no pairing is possible for quarks with momenta between the  $u$  and  $s$  Fermi momenta, and that at most one of the quarks in a  $u$ - $s$  Cooper pair can be at its respective Fermi surface. The BCS singularity therefore does not arise if  $M_s \neq 0$ , and a  $u$ - $s$  condensate is not guaranteed. A  $u$ - $s$  condensate involves pairing of quarks with momenta within about  $\Delta_{us}$  of the Fermi surface, and we therefore expect that  $\Delta_{us}$  can only be nonzero if the mismatch between the up and strange Fermi momenta is less than or of order  $\Delta_{us}$ :

$$\sqrt{\mu^2 - M_u(\mu)^2} - \sqrt{\mu^2 - M_s(\mu)^2} \approx \frac{M_s(\mu)^2 - M_u(\mu)^2}{2\mu} \lesssim \Delta_{us} . \quad (4.1)$$

Here  $M_s(\mu)$  and  $M_u(\mu)$  are the constituent quark masses in the CFL phase. We neglect  $M_u(\mu)$  in the following. Equation (4.1) implies that arbitrarily small values of  $\Delta_{us}$  are impossible. As  $m_s$  is increased from zero,  $\Delta_{us}$  decreases until it is comparable to  $M_s(\mu)^2/2\mu$ . At this point, smaller nonzero values of  $\Delta_{us}$  are not possible, and  $\Delta_{us}$  must therefore vanish discontinuously. This simple physical argument leads us to conclude that the unlocking phase transition at  $M_s = M_s^{\text{unlock}}$  must be first order. This was confirmed by explicit calculation in Ref. [14], where the physics of the quark matter phases in Figure 3 was analyzed in the mean-field approximation in a toy model, where the full interactions between quarks were replaced by a four-fermion interaction with the quantum numbers of single-gluon exchange.

## 5 The 2+1 flavor quark pairing ansatz

To analyze the phase diagram one must allow for at least three condensates, two in quark pair channels and one in the chiral channel,

$$\langle q_i^\alpha C \gamma_5 q_j^\beta \rangle , \quad \langle q_i^\alpha C \gamma_5 \gamma_4 q_j^\beta \rangle , \quad \langle \bar{q}_\alpha^i q_j^\beta \rangle , \quad (5.1)$$

leading to gap parameters

$$\Delta_{ij}^{\alpha\beta}, \quad \kappa_{ij}^{\alpha\beta}, \quad \phi_{\alpha j}^{i\beta} \quad (5.2)$$

Each gap matrix is a symmetric  $9 \times 9$  matrix describing the color (Greek indices) and flavor (Roman indices) structure.

In Ref. [14] we made several simplifying assumptions to obtain easily soluble gap equations. These are: (1) Fix  $\phi_{\alpha j}^{i\beta} = M_s \delta_3^i \delta_j^3 \delta_\alpha^\beta$ . (2) Use the simplest form for  $\Delta_{ij}^{\alpha\beta}$  which allows an interpolation between the color-flavor locking favored by single-gluon exchange at  $m_s = 0$  and the “2SC” phase favored at  $m_s \rightarrow \infty$  (see (5.6)). This ansatz, which requires five independent superconducting gap parameters, leads to consistent gap equations in the presence of the one-gluon exchange interaction. (3) Neglect  $\kappa_{ij}^{\alpha\beta}$ . This condensate pairs left-handed and right-handed quarks, and so breaks chiral symmetry. Its effects are small [14].

With these simplifying assumptions, the simplest ansatz that interpolates between the two flavor case ( $m_s = \infty$ ) and the three flavor case ( $m_s = 0$ ) is

$$\Delta_{ij}^{\alpha\beta} = \begin{pmatrix} b+e & b & c & & & & & & \\ & b & b+e & c & & & & & \\ & c & c & d & & & & & \\ & & & & e & & & & \\ & & & e & & & & & \\ & & & & & f & & & \\ & & & & & f & & & \\ & & & & & & f & & \\ & & & & & & & f & \end{pmatrix}$$

basis vectors:

$$\begin{aligned} (\alpha, i) &= (1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2) \\ &= (r, u), (g, d), (b, s), (r, d), (g, u), (r, s), (b, u), (g, s), (b, d) \end{aligned} \quad (5.6)$$

where the color indices are  $\alpha, \beta$  and the flavor indices are  $i, j$ . The strange quark is  $i = 3$ . The rows are labelled by  $(\alpha, i)$  and the columns by  $(\beta, j)$ .

The properties of the ansatz are summarized in Table 2. In its general form, this condensate locks color and flavor. This is because of the condensates  $c$  and  $f$ , referred to collectively as  $\Delta_{us}$  above, that combine a strange quark with a light one. It is straightforward to confirm by direct calculation that if either  $c$  or  $f$  or  $b + e$  is nonzero, then the matrix  $\Delta_{ij}^{\alpha\beta}$  of (5.6) is not invariant under separate flavor or color rotations but is left

description	condensate	symmetry
2SC: 2-flavor superconductivity	$c = d = f = 0,$ $b = -e$	$SU(2)_L \times SU(2)_R$
CFL: color-flavor locking		$SU(2)_{\text{color}+L+R}$
CFL: color-flavor locking with $m_s = 0$	$c = b, f = e,$ $d = b + e$	$SU(3)_{\text{color}+L+R}$

Table 2: Symmetries of the condensate ansatz (5.6) in various regimes.

invariant by simultaneous rotations of  $SU(2)_V$  and the  $SU(2)$  subgroup of color corresponding to the colors 1 and 2. Thus, color-flavor locking occurs whenever one or more of  $c$ ,  $f$ , or  $b + e$  is nonzero.

Although the standard electromagnetic symmetry is broken in the CFL phase, as are all the color gauge symmetries, there is a combination of electromagnetic and color symmetry that is preserved [10] (see Table 2). Consider the gauged  $U(1)$  under which the charge  $\tilde{Q}$  of each quark is the sum of its electromagnetic charge  $(2/3, -1/3, -1/3)$  (depending on the flavor of the quark) and its color hypercharge  $(-2/3, 1/3, 1/3)$  (depending on the color of the quark). It is easy to confirm that the sum of the  $\tilde{Q}$  charges of each pair of quarks corresponding to a nonzero entry in (5.6) is zero. This modified electromagnetism is therefore not broken by the condensate (see Section 8).

## 6 Quark-hadron continuity

As has been emphasized above, for low enough  $m_s$  the CFL phase may consist of hadronic matter at low  $\mu$ , and quark matter at high  $\mu$ . This raises the possibility [15] that properties of sufficiently dense hadronic matter could be found by extrapolation from the quark matter regime where models like the one considered in this paper can be used as a guide at moderate densities, and where the QCD gauge coupling becomes small at very high densities.

The most straightforward application of this idea is to relate the quark/gluon description of the spectrum to the hadron description of the spectrum in the CFL phase [15]. As  $\mu$  is decreased from the regime in which a quark/gluon description is convenient to one in which a baryonic description

Quark	$SU(2)_{\text{color}+V}$	$\tilde{Q}$	gap	Hadron	$SU(2)_V$	$Q$	gap
$\begin{pmatrix} bu \\ bd \end{pmatrix}$	<b>2</b>	+1	$f$	$\begin{pmatrix} p \\ n \end{pmatrix}$	<b>2</b>	+1	$\Delta_4^B$
$\begin{pmatrix} gs \\ rs \end{pmatrix}$	<b>2</b>	0		$\begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}$	<b>2</b>	0	
		-1				-1	
$\begin{pmatrix} ru - gd \\ gu \\ rd \end{pmatrix}$	<b>3</b>	0	$e$	$\begin{pmatrix} \Sigma^0 \\ \Sigma^+ \\ \Sigma^- \end{pmatrix}$	<b>3</b>	0	$\Delta_3^B$
		+1				+1	
		-1				-1	
$ru + gd + \xi_- bs$	<b>1</b>	0	$\Delta_-$	$\Lambda$	<b>1</b>	0	$\Delta_1^B$
$ru + gd - \xi_+ bs$	<b>1</b>	0	$\Delta_+$	—			

$$\Delta_{\pm} = \frac{1}{2} \left( 2b + e + d \pm \sqrt{(2b + e - d)^2 + 8c^2} \right)$$

$$\xi_{\pm} = -\frac{1}{2c} \left( 2b + e - d \mp \sqrt{(2b + e - d)^2 + 8c^2} \right)$$

Table 3: Comparison of states and gap parameters in high density quark and hadronic matter.

is convenient, there is no change in symmetry so there need be no transition: the spectrum of the theory may change continuously. Under this mapping, the massive gluons in the CFL phase map to the octet of vector bosons;<sup>1</sup> the Goldstone bosons associated with chiral symmetry breaking in the CFL phase map to the pions; and the quarks map onto baryons. Pairing occurs at the Fermi surfaces, and we therefore expect the gap parameters in the various quark channels to map to the gap parameters due to baryon pairing.

In Table 3 we show how this works for the fermionic states in 2+1 flavor QCD. There are nine states in the quark matter phase. We show how they transform under the unbroken “isospin” of  $SU(2)_{\text{color}+V}$  and their charges under the unbroken “rotated electromagnetism” generated by  $\tilde{Q}$ , as described above. Table 3 also shows the baryon octet, and their transformation

<sup>1</sup>The singlet vector boson in the hadronic phase does not correspond to a massive gluon in the CFL phase. This has been discussed in Ref. [15].



properties under the symmetries of isospin and electromagnetism that are unbroken in sufficiently dense hadronic matter. Clearly there is a correspondence between the two sets of particles.

(The one exception is the final isosinglet, discussed at greater length in Ref. [14]. In the  $\mu \rightarrow \infty$  limit, where the full 3-flavor symmetry is restored, it becomes an  $SU(3)$  singlet, so it is not expected to map to any member of the baryon octet. The gap  $\Delta_+$  in this channel is twice as large as the others—it corresponds to  $\Delta_1$  in Ref. [10].)

When we map the quark states onto baryonic states, we can predict that the baryonic pairing scheme that will occur is the one conjectured in Sect. 1 for sufficiently dense baryonic matter:

$$\begin{aligned} \langle p\Xi^- \rangle, \langle \Xi^- p \rangle, \langle n\Xi^0 \rangle, \langle \Xi^0 n \rangle &\rightarrow 4 \text{ quasiparticles, with gap parameter } \Delta_4^B \\ \langle \Sigma^+ \Sigma^- \rangle, \langle \Sigma^- \Sigma^+ \rangle, \langle \Sigma^0 \Sigma^0 \rangle &\rightarrow 3 \text{ quasiparticles, with gap parameter } \Delta_3^B \\ \langle \Lambda\Lambda \rangle &\rightarrow 1 \text{ quasiparticle, with gap parameter } \Delta_1^B \end{aligned} \quad (6.1)$$

The baryon pairs are rotationally-invariant,  $Q$ -neutral,  $SU(2)_V$  singlets. It seems reasonable to conclude that as  $\mu$  is increased the baryonic gap parameters  $(\Delta_4^B, \Delta_3^B, \Delta_1^B)$  may evolve continuously to become the quark matter gap parameters  $(f, e, \Delta_-)$ .

It should be born in mind, however, that the physical quantities are the gaps. What we have discussed above are the gap parameters, or condensates (technically, the 1PI two-point functions). The gaps are a function of these and the particle masses. Since the masses are very different in the nuclear and quark phases, the gaps may be quite different too, even if the gap parameters are similar [22].

## 7 Color-Flavor Locking at Asymptotic Densities

At asymptotically high densities, the QCD coupling  $g$  is weak at the Fermi surface, so diagrammatic methods can be used [4, 23, 24, 25] to determine the leading behavior of the gap.

$$\Delta \sim C\mu \frac{1}{g(\mu)^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}} \frac{1}{g(\mu)}\right), \quad (7.1)$$

It is striking that when these calculations are extrapolated to low density, they give gaps of order 100 MeV, in agreement with NJL calculations [24, 25].

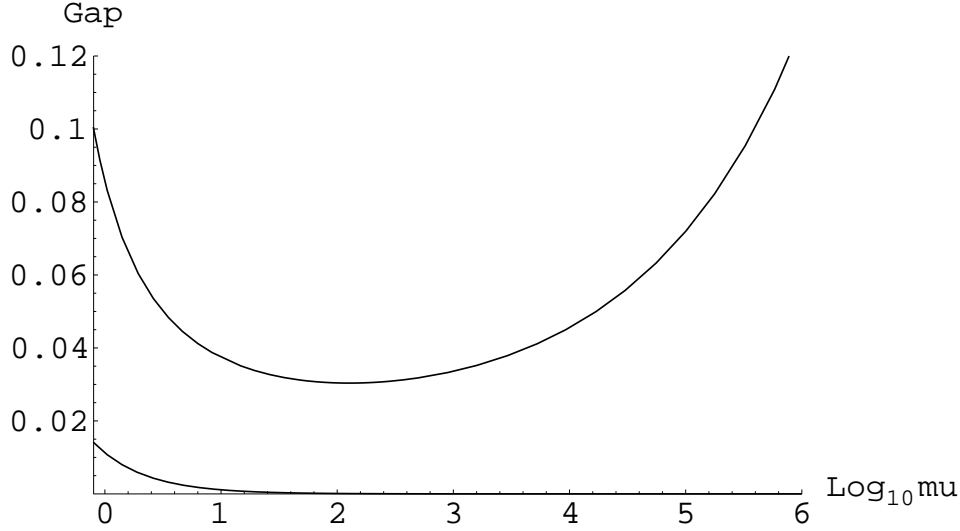


Figure 5: The upper curve shows the weak-coupling QCD result for the superconducting gap  $\Delta$  as a function of  $\log_{10} \mu$ , for  $\mu$  from 0.8 GeV to  $10^6$  GeV. The vertical scale has been normalized so that  $\Delta = 0.1$  GeV at  $\mu = 0.8$  GeV. We have taken  $g(\mu)$  from the two-loop beta function for three flavor QCD with  $\Lambda_{\text{QCD}} = 200$  MeV. Color-flavor locking occurs whenever  $\Delta \gtrsim M_s^2/2\mu$ . The lower curve is  $M_s^2/2\mu$ , taking  $M_s = 150$  MeV. We conclude that QCD at very high densities is in the CFL phase.

We have therefore fixed the prefactor  $C$  so that  $\Delta = 100$  MeV at  $\mu = 800$  MeV. We show the result in Figure 5. Note that  $\Delta$  is plotted versus  $\log \mu$ ; it changes very slowly. It decreases by about a factor of three as  $\mu$  is increased to around 100 GeV and then begins to rise without bound at even higher densities. This is basically because  $\mu$  rises faster than  $\exp(-1/g)$  drops. We conclude that independent of any details (like the precise value of  $M_s$ , for example) at asymptotically high densities  $\Delta$  is far above  $M_s^2/2\mu$ . For any finite value of the strange quark mass  $m_s$ , quark matter is in the color-flavor locked phase, with broken chiral symmetry, at arbitrarily high densities where the gauge coupling becomes small.

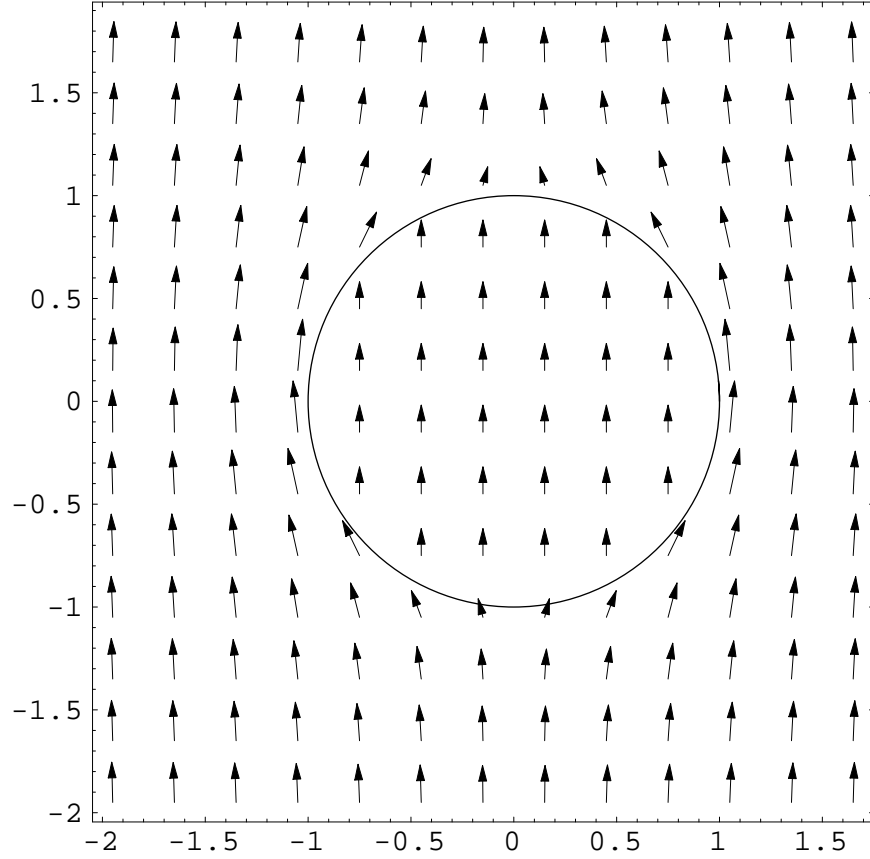


Figure 6: Locally unbroken magnetic field inside and outside a sphere of high-density quark matter, with sharp boundary.

## 8 A signature for color superconducting neutron stars

The most likely place to find superconducting quark phases in nature is in the core of neutron stars. If neutron stars achieve sufficient central densities that they develop quark matter cores, these cores must be color superconductors (see Section 1). It is therefore useful to analyze the physical properties of dense quark matter and try to give observable signatures of the color superconducting phase, thus allowing astrophysical observation to play a role.

Though color and ordinary electromagnetism are broken in a color

superconductor, there is a linear combination of the photon and a gluon that remains massless (Section 5). Consequently, a color superconducting region may be penetrated by an external magnetic field. In Ref. [26] it was shown that at most a small fraction of the magnetic field is expelled, and if the screening distance is the smallest length scale in the problem there is no expulsion at all. It was found that color-superconducting regions in a neutron star core would admit magnetic fields without restricting them to quantized flux tubes. Such magnetic fields are stable on time scales longer than the age of the universe, even if the spin period of the neutron star is changing.

This is interesting because it interferes with the main mechanism by which the magnetic fields of isolated pulsars are supposed to decay. This is the dragging of flux tubes by rotational vortices. As the star spins down, its rotational vortices move outwards, dragging the flux tubes with them. When the flux tubes reach the crust, they decay away. The conductivity in the crust gives a decay time of  $10^6$ - $10^{10}$  years [27].

In contrast, we would predict that as the spin period of the neutron star changes and the rotational vortices move accordingly, there is no change at all in the strength of the  $\tilde{Q}$ -magnetic field in the core. The data on isolated pulsars indicates that the decay time is  $10^8$  years or more [28], and is therefore consistent with our hypothesis.

## 9 Conclusions

We have discussed a conjectured phase diagram for  $2 + 1$  flavor QCD as a function of  $\mu$ , the chemical potential for quark number, and  $m_s$ , the strange quark mass. We have worked at zero temperature and ignored electromagnetism and the  $u$  and  $d$  quark masses throughout. The phases may be summarized as follows (see Fig. 3).

- There are basically two types of quark matter:
  - CFL: *color-flavor locking* quark matter; all 3 flavors participate, breaking chiral symmetry.
  - 2SC: *two-flavor color superconducting* quark matter;  $u$ - $d$  pairing, chirally symmetric, *non*-superfluid.
- The phase transition between CFL and 2SC is first-order, assuming no other phase intervenes.

- If  $m_s$  is low enough, then as density rises there is a transition from baryonic matter to chirally broken CFL quark matter. In this case,
  - Quark matter and baryonic matter may be continuously connected: symmetries do not require any phase transition between them.
  - Chiral symmetry is broken at all densities.
- If  $m_s$  is high enough, then as density rises there is a transition from baryonic matter to 2SC (chirally restored) quark matter, then to CFL.
- At sufficiently high density, chiral symmetry is always broken.

There are many directions in which this work can be developed. We have worked at zero temperature, so a natural extension would be to study the effects of finite temperature. The phase diagram of two-flavor QCD as a function of baryon density, temperature and quark mass has been explored in Ref. [8]. It would be interesting to perform a similar study of the 3 flavor and 2+1 flavor cases, building on the suggestions of Pisarski [29].

Within NJL models, one could study more exotic channels such as those with  $S = 1$  and/or  $L = 1$ , or channels that would lead to pairing of the strange quarks in the 2SC+s phase. A particularly intriguing possibility is that there is a crystalline phase between the CFL and 2SC phases. Such phases have been posited in condensed matter contexts [12], and are currently being studied in quark matter [13].

Great progress has been made during the last year in using weak-coupling methods, appropriate to the limit of asymptotically high density, to calculate the parameters of the effective theory for the light degrees of freedom at the Fermi surface [30]. But there remains disagreement over the behavior of the masses of the pseudo-Goldstone bosons in the large- $\mu$  limit. It would also be interesting to try to include the effects of instantons in order to help with the extrapolation to the moderate density regime that is of physical interest.

It is of great importance to continue to investigate possible observable consequences of the color-superconducting state. The natural arena is the phenomenology of neutron/quark stars, which are the only naturally occurring example of cold matter at the densities we have studied. We have already discussed possible effects of the “rotated” electromagnetism on magnetic fields. One also expects that the gaps in the quasiquark spectrum will affect cooling by neutrino emission, and the shear and bulk viscosities,

which play an important role in the  $r$ -mode spin-down mechanism [31]. Finally, we should not forget that although current heavy-ion experiments are oriented primarily towards producing hot low-density fireballs, it is possible that color-superconducting phases can be made in the cooler conditions of lower-energy collisions. It would be valuable to investigate possible signatures: the effect on strangeness production of the quark-pairing contribution to gluon mass is one obvious possibility.

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